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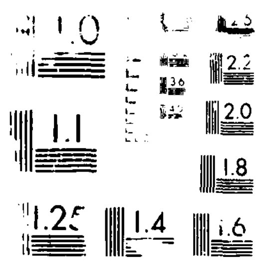
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by

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SUMMARY

Cosmos 1335 (1982-07A) was launched on 29 January 1982 into a near-circular orbit of inclination 74°, and decayed on 5 April 1987. The orbit has been determined from observations for 26 epochs between September 1985 and May 1986, a time when the orbit was experiencing the effects of 31:2 resonance with the Earth's gravitational field. About 1400 observations were used, the most numerous being those from the US Navy Navspasur system, and the most accurate those from the Hewitt cameras of the University of Aston sited at Herstmonceux and at Siding Spring in Australia. The average orbital accuracy achieved was about 70 m radial and 120 m cross-track.

Analysis of the changes in inclination and eccentricity at resonance has yielded useful values for six lumped harmonics in the geopotential of order 31: the two lumped harmonics of even degree had an average accuracy equivalent to 1.5 cm in geoid height; the four of odd degree had accuracies between 2 and 7 cm.

Departmental Reference: Special Systems 2

LIST OF CONTENTS

TR 82041

1 INTRODUCTION

The satellite 1982-07A, Cosmos 1335, was launched on 29 January 1982 and decayed on 5 April 1987. It is believed[1] to have been an octagonal ellipsoid in shape, of length about 1.8 m and diameter about 1.5 m; its mass was about 550 kg. The initial perigee and apogee heights were 482 and 518 km respectively, the initial orbital period was 94.63 minutes, and the orbital inclination 74.0°.

In November 1985 Cosmos 1335 passed through the condition of 31:2 resonance, when the track over the Earth repeats every two days after 31 revolutions. The aim of this Report is to compute accurate orbits from observations during the time when the 31:2 resonance with the geopotential was affecting the orbit, and then to evaluate lumped geopotential harmonics of order 31 from the changes they produced in the orbital inclination and eccentricity. The orbit was determined between September 1985 and May 1986 from radar and optical observations using the RAE orbit refinement program PROP, in the PROP6 version[2].

2 THE OBSERVATIONS, ORBITS AND OBSERVATIONAL ACCURACY

2.1 The observations

The orbit of 1982-07A has been determined at 26 epochs from 1398 observations between 23 September 1985 and 7 May 1986. Table 1 gives the number of observations accepted for each orbit and their sources.

Observations came from three different sources, the most accurate being those from the University of Aston's Hewitt cameras at the RGO site at Herstmonceux (marked H in Table 1) and at the Siding Spring site in Australia (marked A). These observations were available for 6 of the 26 orbits and usually have an accuracy of a few seconds of arc.

The second group consisted of 35 visual observations made by volunteer observers who report to the University of Aston, and these observations usually have accuracies between 2 and 5 minutes of arc.

The third and largest group of observations comprised Navspasur observations kindly supplied by the US Naval Research Laboratory, usually with accuracies of about 2 minutes of arc. There were 1317 of these observations available.

2.2 The orbits

The orbits were determined at approximately 8-day intervals from September 1985 to May 1986 with the aid of the RAE orbit refinement program in the PROP6 version, and the orbital elements at the 26 epochs are listed in

Table 1

Sources of the observations accepted in the final orbits

Orbit number	Source of observation				Total
	Hewitt H	camera A	Visual	US Navy	
1			3	61	64
2			2	34	36
3				18	18
4				36	36
5				55	55
6				46	46
7	8		1	42	51
8	4	2	6	49	61
9			2	46	48
10				48	48
11				37	37
12		4		56	60
13			4	43	47
14				74	74
15	7	4	7	36	54
16				53	53
17				50	50
18				80	80
19				57	57
20		4		54	58
21				47	47
22			4	62	66
23				60	60
24			2	50	52
25	9	4	4	57	74
26				66	66
Total	28	18	35	1317	1398

Table 2

Values of orbital parameters at 26 epochs with standard deviations

	MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	M_4	ε	N	D	a(1 − e)
1	46331	1985 Sep 23	6758.6417 6	0.000603 12	74.0325 11	282.626 1	7.4 5	95.6 5	5626.5008 7	0.0478 1	−0.00062 7	−	0.59	64	7.7	6754.57
2	46339	Oct 1	6758.0806 5	0.000281 16	74.0295 12	264.724 1	342.6 2.1	115.1 2.1	5627.2016 6	0.0382 5	−	−	0.54	36	5.8	6756.18
3	46346	Oct 8	6757.4604 10	0.000108 21	74.0338 21	249.056 2	237.2 10.0	355.6 10.0	5627.9769 13	0.0491 9	−	−	0.45	18	4.0	6756.73
4	46360	Oct 22	6756.2158 5	0.000722 18	74.0382 12	217.716 1	169.6 7	349.4 7	5629.5330 7	0.0645 1	0.00022 4	−	0.59	36	9.5	6751.34
5	46368	Oct 30	6755.4498 4	0.001049 17	74.0365 10	199.801 1	154.8 4	24.2 4	5630.4908 5	0.0509 2	−0.00092 8	−	0.59	55	6.5	6748.36
6	46376	Nov 7	6754.7944 3	0.001332 12	74.0357 11	181.876 1	143.8 3	62.0 3	5631.3105 3	0.0488 1	−	−	0.47	46	7.5	6745.80
7	46382	Nov 13	6754.3117 4	0.001481 5	74.0323 7	168.428 1	135.8 5	4.5 5	5631.9142 5	0.0572 6	0.00467 35	−	0.83	51	3.5	6744.31
8	46387	Nov 18	6753.7747 8	0.001640 7	74.0321 6	157.213 1	128.2 3	80.6 3	5632.5861 10	0.0667 3	0.00127 20	−	0.68	61	6.0	6742.70
9	46394	Nov 25	6753.0918 6	0.001770 8	74.0283 9	141.512 1	116.9 3	265.4 3	5633.4406 7	0.0504 2	−0.00083 9	−	0.43	48	7.2	6741.14
10	46406	Dec 7	6751.9164 3	0.001917 9	74.0294 13	114.578 1	101.4 5	18[illegible] 5	5634.9124 4	0.0532 1	−	−	0.59	48	10.4	6738.97

Table 2 (continued)

MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	M_4	[illegible]	N	D	a(1 − e)
11 [illegible]	1985 Dec 19	6750.6187 13	0.0019[illegible]8 8	74.0330 11	87.628 1	84.4 8	115.4 8	5636.5381 17	0.0729 3	0.00136 13	–	0.75	37	9.4	6737.74
12 [illegible]	Dec 28	6749.75[illegible]4 5	0.0018[illegible]4 6	74.0355 6	67.[illegible] 1	72.7 2	78.6 2	5637.6264 7	0.0556 1	0.00069 6	–	0.57	60	7.5	6737.57
13 [illegible]	1986 Jan 5	6749.1030 10	0.001677 20	74.0330 14	49.423 1	62.4 5	172.9 5	5638.4377 12	0.0389 8	0.00117 22	0.00036 7	0.65	47	7.0	6737.78
14 [illegible]	Jan 13	6748.4015 5	0.001504 6	74.0294 6	31.433 1	52.8 3	273.2 3	5639.317[illegible] 6	0.0485 1	−0.00050 6	–	0.43	74	7.4	6738.25
15 [illegible]	Jan 25	6747.3264 3	0.001121 6	74.0317 7	4.429 <1	36.6 3	258.[illegible] 3	5640.6635 4	[illegible]595 2	–	–	0.44	54	6.1	6739.76
16 [illegible]	Feb 2	6745.5261 2	0.000819 11	74.0336 11	346.426 1	30.8 6	12.9 6	5641.6698 2	[illegible] 1	–	–	0.58	53	8.0	6741.00
17 [illegible]	Feb 13	6744.4899 4	0.000372 20	74.0361 17	321.648 1	23.4 1.5	142.9 1.5	5642.[illegible] 5	[illegible]887 2	–	–	0.69	[illegible]	7.0	6741.98
18 [illegible]	Feb 24	6743.3449 4	0.000144 8	74.0325 9	296.851 1	127.0 4.1	186.3 4.1	5643.6638 6	[illegible] 3	[illegible] 3	[illegible] 1	[illegible]	80	9.6	6742.37
19 [illegible]	Mar 4	6742.3648 5	0.000427 13	74.0291 12	278.806 1	150.8 1.1	312.0 1.1	5646.8951 7	[illegible]825 3	[illegible] 14	–	[illegible]62	[illegible]	[illegible]	6739.48
20 [illegible]	Mar 14	6740.7627 6	0.000796 11	74.0279 11	256.231 1	147.3 6	249.1 6	5648.9091 7	0.0884 2	−0.00194 6	–	0.85	58	10.0	6735.40
[illegible]	Mar [illegible]	[illegible]	[illegible]	74.0287 8	233.636 1	137.0 7	210.4 7	5650.4101 8	0.0880 1	0.00148 6	–	0.59	47	8.4	6732.70

Table 2 (concluded)

	MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	M_4	ε	N	D	a(1 - e)
22	46522	1986 Apr 2	6738.2471 4	0.001239 7	74.0322 9	213.294 1	126.1 4	299.6 4	5652.[illegible]743 4	0.0863 1	-0.00043 3	-	0.55	66	9.4	6729.90
23	46532	Apr 12	6736.8935 3	0.001437 7	74.0333 8	19[illegible].675 1	114.7 7	294.4 7	5653.7785 4	0.0912 1	-0.00117 2	-	0.67	60	9.9	6727.21
24	46543	Apr 23	6735.5041 2	0.001530 5	74.0290 7	165.779 1	103.5 6	198.8 6	5655.5283 3	0.0772 3	-	-	0.55	52	5.9	6725.20
25	46549	Apr 29	6734.7633 7	0.001483 4	74.0305 4	152.186 <1	97.5 1	285.5 1	5656.4620 9	0.0738 6	0.00011 13	-0.00023 7	0.63	74	6.0	6724.78
26	46557	May 7	6733.5685 5	0.001529 5	74.0252 10	134.066 1	89.7 5	170.0 5	5657.9678 6	0.1149 7	-0.00353 13	-0.00219 9	0.64	66	5.4	6723.27

Key:
- MJD = modified Julian day
- a = semi major axis (km)
- e = eccentricity
- i = inclination (deg)
- Ω = right ascension of ascending node (deg)
- ω = argument of perigee (deg)
- M_0 = mean anomaly at epoch (deg)
- M_1 = mean motion n (deg/day)
- M_2-M_4 = later coefficients in the polynomial for M
- ε = measure of fit
- N = number of observations accepted
- D = time covered by the observations (days)

Table 2, with the standard deviations below each value. The epoch for each orbit is at 00 hours on the day indicated, and the PROP program fits the mean anomaly M by a polynomial of the form

$$M = M_0 + M_1 t + M_2 t^2 + M_3 t^3 + M_4 t^4 + M_5 t^5 \quad , \tag{1}$$

where t is the time measured from epoch and the number of M coefficients used depends on the drag. For 1982-07A, which was in a near-circular orbit at a height of about 370 km, M_0-M_2 were the only coefficients needed on 8 orbits; 14 needed up to M_3, and 4 required M_0-M_4.

The value of ε, the parameter indicating the measure of fit of the observations to the orbit, varied between 0.43 and 0.85, and had an average value of 0.60. The 6 orbits with Hewitt camera observations are generally more accurate than the rest: for inclination the average standard deviation for the 6 Hewitt camera orbits is 0.0007° and for the other orbits 0.0011°; for the eccentricity the average standard deviations are 0.000006 and 0.000011, respectively. For all 26 orbits the average standard deviation in inclination is 0.0010°, which is equivalent to about 120 m in distance, and 0.000010 in eccentricity, equivalent to 70 m in distance.

Fig 1 shows the PROP values of inclination with their standard deviations.

The values of eccentricity are plotted in polar form in Fig 2. This diagram is interesting because the angular motion of perigee changes from circulation through 360° initially to libration about $\omega = 90°$ later. This happens because the circular locus of the perigee under the action of odd zonal harmonics is converted into a spiral by the effects of air drag (and possibly resonance). The locus initially passes below the origin, but on its next crossing of the $e \sin \omega$ axis it is above the origin.

2.3 The accuracy of the observations

The residuals of the observations have been printed out with the ORES computer program[3], and have been sent to the observers. The rms residuals for observing stations with 3 or more observations accepted in the orbit determination are given in Table 3. The US Navy observations from station 29 are geocentric, and if they were given in the same form as the topocentric observations their angular rms residuals would increase by a factor between 5 and 10. In calculating the rms for the visual observers, residuals greater than twice the rms have been omitted (the numbers used being shown in brackets).

Table 3

Residuals for observing stations with 3 or more observations accepted in the final orbit determinations

Station		Number of observations		rms residuals			
				Range km	Minutes of arc		
		Accepted	Rejected		RA	Dec	Total
1	US Navy	215	9		2.4	2.6	3.6
2	US Navy	156	23		2.8	3.9	4.8
3	US Navy	198	18		3.2	3.2	4.5
5	US Navy	162	12		3.8	2.5	4.5
6	US Navy	185	12		1.9	2.6	3.2
29	US Navy	401	9	0.5	0.2*	0.5*	
2265	Farnham	5 (4)	0		3.4	4.9	6.0
2414	Bournemouth	3	0		4.2	1.7	4.5
2418	Sunningdale	7 (6)	1		2.7	4.6	5.3
2420	Willowbrae	15 (12)	2		4.0	3.4	5.2
2659	Herstmonceux 3 (Hewitt camera)	28	0		0.06	0.07	0.09
9652	Siding Spring (Hewitt camera)	18	0		0.03	0.04	0.05

* geocentric

The residuals in Table 3 are rather larger than usual, mainly because the orbit was much closer to the Earth than in most previous orbit determinations of this type, the height (over the equator) being between 345 and 390 km. Consequently the satellite could often only be seen at low elevation, where observations tend to be less accurate; this especially affects the US Navy observations. When the satellite was observed at high elevation, it crossed the sky at a rapid angular rate, thus aggravating the effects of timing errors, particularly by visual observers. Also the satellite was small, and too faint to be ideal for visual and photographic observing.

In view of these difficulties, the overall rms residuals of the Hewitt cameras are remarkably good, being 5 seconds of arc for the Herstmonceux camera and 3 seconds of arc for the Siding Spring camera. Since the residuals combine the orbital and observational errors, the observational errors of the Hewitt cameras are likely to be smaller than their rms residuals, and 2 seconds of arc would be an accuracy consistent with the results.

3 THEORY FOR THE RESONANCE EFFECTS

This theory has often been given in detail before (for example in Ref 4), and will only be summarized here. The longitude-dependent part of the geopotential at an exterior point (r,θ,λ) is written as[5]

$$\frac{\mu}{r}\sum_{\ell=2}^{\infty}\sum_{m=1}^{\ell}\left(\frac{R}{r}\right)^{\ell} P_{\ell}^{m}(\cos\theta)\left\{\bar{C}_{\ell m}\cos m\lambda + \bar{S}_{\ell m}\sin m\lambda\right\} N_{\ell m} \quad , \qquad (2)$$

where r is the distance from the Earth's centre, θ is co-latitude, λ is longitude (positive to the east), μ is the gravitational constant for the Earth (398600 km^3/s^2), R is the Earth's equatorial radius (6378.1 km), $P_{\ell}^{m}(\cos\theta)$ is the associated Legendre function of order m and degree ℓ, and $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ are the normalized tesseral harmonic coefficients, of which only those of order $m = 31$ are relevant here. The normalizing factor $N_{\ell m}$ is given by

$$N_{\ell m}^{2} = \frac{2(2\ell+1)(\ell-m)!}{(\ell+m)!} \quad . \qquad (3)$$

The rate of change in inclination i caused by a relevant pair of coefficients, $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$, near $\beta:\alpha$ resonance may be written[4] (ignoring terms of order e^2) as

$$\frac{di}{dt} = \frac{n}{\sin i}\left(\frac{R}{a}\right)^{\ell}\bar{F}_{\ell mp}G_{\ell pq}(k\cos i - m)\Re\left[j^{\ell-m+1}(\bar{C}_{\ell m} - j\bar{S}_{\ell m})\exp\{j(\gamma\Phi - q\omega)\}\right] \quad , \qquad \ldots\ldots (4)$$

where $\bar{F}_{\ell mp}$ is Allan's normalized inclination function[6], $G_{\ell pq}$ is a function of eccentricity e for which explicit forms have been derived by Gooding, $\Re$ denotes 'real part of' and $j = \sqrt{-1}$. The resonance angle Φ is defined by the equation

$$\Phi = \alpha(\omega + M) + \beta(\Omega - \nu) \quad , \qquad (5)$$

where ω is the argument of perigee, M the mean anomaly, Ω the right ascension of the node and ν the sidereal angle. The indices γ, q, k and p in equation (4) are integers, with γ taking the values 1, 2, 3 .... and q the values 0, ±1, ±2, ...; the equations linking ℓ, m, k and p are: $m = \gamma\beta$; $k = \gamma\alpha - q$; $2p = \ell - k$.

Here $\beta = 31$ and $\alpha = 2$, and we shall ... terms, which are usually dominant. The values of ℓ to be ... be such that $\ell \geq m$ and $(\ell - k)$ is even. The successive coefficients which arise (for given γ and q) may be grouped into a lumped harmonic, written as[4]

$$\bar{C}_m^{q,k} = \sum_{\ell} Q_{\ell}^{q,k} \bar{C}_{\ell m} , \qquad \bar{S}_m^{q,k} = \sum_{\ell} Q_{\ell}^{q,k} \bar{S}_{\ell m} , \tag{6}$$

where ℓ increases in steps of 2 from its minimum permissible value ℓ_0, and the Q_ℓ are constant coefficients, with $Q_{\ell 0} = 1$. The values of the Q_ℓ can be obtained from equation (4), and R.H. Gooding has written a computer program PROF for their evaluation.

It has often been shown (for example in Ref 7) that, in the equation (4) for di/dt, only the $q = 0$ terms are important if e is small. If $q = 0$ and $\gamma = 1$, then $k = \alpha = 2$ so that the relevant lumped harmonics for di/dt are $\bar{C}_{31}^{0,2}$ and $\bar{S}_{31}^{0,2}$. Since $(\ell - k)$ must be even, the values of ℓ in the summations in equation (6) are 32,34,36, ... The numerical values of the Q_ℓ for 1982-07A are given in section 4.

The rate of change of eccentricity produced by a ... pair of coefficients $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ near $\beta:\alpha$ resonance may be written

$$\frac{de}{dt} = n\left(\frac{R}{a}\right)^{\ell} \bar{F}_{\ell mp} G_{\ell pq} \left\{\frac{q - \frac{1}{2}(k + q)e^2}{e}\right\} \Re\left[j^{\ell-m+1} (\bar{C}_{\ell m} - j\bar{S}_{\ell m}) \exp j(\gamma\Phi - q\omega)\right] , \tag{7}$$

where terms of order e^2 have again been ignored.

For near-circular orbits, the only important terms in (7) are those with $q = 1$ and $q = -1$, because all the others are multiplied by powers of e. With $\alpha = 2$ and $\gamma = 1$, the value of $k(= \gamma\alpha - q)$ is $2 - q$: thus $(q,k) = (1,1)$ and $(-1,3)$ are the important terms. Since $(\ell - k)$ must be even, the values of ℓ arising in the summations of equation (6) are 31,33,35 ... . Numerical values for 1982-07A are given in section ...

4 ANALYSIS OF INCLINATION

4.1 The fitting

Before the effect of resonance can be analysed, ... inclination must be cleared of other perturbations. The perturbations ... zonal harmonics and lunisolar gravitational effects have been removed by use of the PROD computer program[8] with integration at 1-day intervals. The perturbations due to the $J_{2,2}$ harmonic are recorded with each PROP run and have also been removed. The effect of air drag was removed within the THROE computer program[9], assuming[10] an atmospheric rotation rate Λ of 1.1 rev/day. Other perturbations, such as solar radiation pressure, and earth and ocean tides, have been ignored, as they are expected to be less than the standard deviations of the values.

Although the raw values of inclination (Fig 1) look unpromising for analysis, the corrected values, shown in Fig 3, are much more regular, and a theoretical curve from integration of equation (4) was fitted with the aid of the THROE computer program[9]. The values of the lumped harmonics obtained from the fitting are:

$$10^9 \bar{C}_{31}^{0,2} = 1.4 \pm 1.7 \quad , \qquad 10^9 \bar{S}_{31}^{0,2} = 11.8 \pm 3.6 \quad . \tag{8}$$

The measure of fit ε had a value of 0.95. (ε^2 is defined as the sum of squares of the weighted residuals, divided by the number of degrees of freedom.) None of the standard deviations needed to be relaxed, but the last three values were omitted from the fitting, because they were a long way past the resonance and thus contributed little, and also because they did not fit well.

Exact resonance ($\dot{\Phi} = 0$) occurred on 18 November 1985, and Fig 4 shows the variation of $\dot{\Phi}$ with time, which is close to a straight line near the resonance.

The fitting in Fig 3 seems entirely satisfactory: the total variation at 31:2 resonance is, as usual, quite small, but the orbits are accurate enough to define the variation fairly well.

4.2 Discussion

The values of the Q coefficients have been calculated with the aid of the PROF computer program, and the lumped harmonics may be expressed in terms of the individual harmonic coefficients of order 31 as follows:

$$\bar{C}_{31}^{0,2} = \bar{C}_{32,31} - 0.060\ \bar{C}_{34,31} - 0.503\ \bar{C}_{36,31} - 0.452\ \bar{C}_{38,31} - 0.181\ \bar{C}_{40,31}$$

$$+ 0.086\ \bar{C}_{42,31} + 0.233\ \bar{C}_{44,31} + 0.242\ \bar{C}_{46,31} + 0.154\ \bar{C}_{48,31} + \ldots \qquad \ldots\ldots (9)$$

The equation for $\bar{S}_{31}^{0,2}$ is similar, with C replaced by S throughout.

If we make the usual assumption that the $\bar{C}_{\ell m}$ are of order $10^{-5}/\ell^2$, all terms in equation (9) of degree ≥40 are less than 12% of the first term. Ignoring these high-degree terms, we can rewrite (9) as

$$\bar{C}_{31}^{0,2}\{1 + O(0.15)\} = \bar{C}_{32,21} - 0.060\,\bar{C}_{34,31} - 0.503\,\bar{C}_{36,31} - 0.452\,\bar{C}_{38,31}\;, \qquad \ldots\ldots (10)$$

with a similar equation for S, where the term O(0.15) represents a conflation of the error terms and is small beside the errors in equation (8), which are greater than 30%.

There have hitherto been only two accurate analyses of 31:2 resonance[11,12], and it so happens that the Q coefficients for one of these, Samos 2 at 98° inclination[12], are not too different from those in equation (10). For Samos 2,

$$\bar{C}_{31}^{0,2} = \bar{C}_{32,31} + 0.257\,\bar{C}_{34,31} - 0.186\,\bar{C}_{36,31} - 0.355\,\bar{C}_{38,31} - \ldots \,. \qquad (11)$$

Thus there is likely to be some similarity between the numerical values of $\bar{C}_{31}^{0,2}$ and $\bar{S}_{31}^{0,2}$ from the two satellites, unless the (34,31), coefficients happen to be large. The values from Samos 2 were: $10^9\bar{C}_{31}^{0,2} = -2.9 \pm 1.2$; $10^9\bar{S}_{31}^{0,2} = 9.0 \pm 2.2$. The values in equation (8) are as close as would be expected to those from Samos 2, and have errors about 50% greater, as is also to be expected because the drag of 1982-07A was double that of Samos 2.

We may roughly estimate the error in geoid height implied by the standard deviations σ in equation (8) as $R\sigma/\bar{Q}$, where R is the Earth's radius and $\bar{Q} = \left\{\sum\left(Q_\ell^{q,k}\ell_0^2/\ell^2\right)^2\right\}^{\frac{1}{2}}$. Here $\bar{Q} = 1.144$, and the errors in geoid height corresponding to the σ in equation (8) are 0.9 cm and 2.1 cm for C and S respectively. The corresponding accuracies from Samos 2 are 0.7 cm and 1.3 cm.

5 ANALYSIS OF ECCENTRICITY

5.1 The fitting

Before analysing the values of e, we need to remove the perturbations due to zonal harmonics and lunisolar gravitational effects, though the latter prove to be very small. This was done with the PROD computer program, using 1-day integration steps (checked by re-running with 0.5-day steps).

The removal of air drag effects within THROE is not satisfactory for 1982-07A, because the air density model within THROE is spherically symmetrical, whereas the real atmosphere has a 'daytime bulge' of high density. In the conditions experienced by 1982-07A, at heights near 350 km with low solar activity, the maximum daytime density is 4 times greater than the minimum night-time density, according to the COSPAR International Reference Atmosphere 1972[15]. The changes in eccentricity produced by an atmosphere of this character were calculated by the method given in the Appendix of Ref 11, and Fig 5 shows the total correction to e, both for a spherically symmetrical atmosphere and for an atmosphere with the day-to-night variation. It will be seen the two are completely different: in a spherical atmosphere, e is always reduced by drag, so that Δe is positive; with a day-to-night variation, the sign of Δe depends on the position of perigee (see equation (9) of Ref 14). After making the atmospheric correction, with day-to-night variation included, the values of e were fitted using THROE with drag set to zero (that is, with $M_2 = 0$).

There were difficulties in fitting the points after MJD 46480: but this was a long way after resonance, as Fig 4 shows, and it was decided to concentrate on the 17 central values. The fitting, with $(\gamma, q) = (1,1)$ and $(1,-1)$ is shown in Fig 6, and the values of lumped harmonics obtained were:

$$\left.\begin{aligned} 10^9\bar{C}_{31}^{1,1} &= 1 \pm 26 , & 10^9\bar{S}_{31}^{1,1} &= -14 \pm 8 \\ 10^9\bar{C}_{31}^{-1,3} &= -41 \pm 10 , & 10^9\bar{S}_{31}^{-1,3} &= -58 \pm 16 \end{aligned}\right\} . \qquad (12)$$

The measure of fit ε had the value of 1.98 and two of the standard deviations were relaxed by a factor of 2, as shown in Fig 6.

The fitting in Fig 6 is reasonably satisfactory, but not as good as might have been hoped, as shown by the rather high value of ε. However, values of ε between 1.5 and 2.0 are quite usual in fittings of e: with Samos 2, for example, $\varepsilon = 1.62$. This probably happens because it is difficult to remove the effects of drag without error.

5.2 Discussion

The equations for the lumped harmonics, with the Q factors calculated with the aid of PROF, are

$$\bar{C}_{31}^{1,1} = \bar{C}_{31,31} - 1.922\,\bar{C}_{33,31} - 0.757\,\bar{C}_{35,31} + 0.558\,\bar{C}_{37,31} + 1.052\,\bar{C}_{39,31}$$

$$+ 0.783\,\bar{C}_{41,31} + 0.169\,\bar{C}_{43,31} - 0.379\,C_{45,31} - \cdots ,$$

$$\ldots\ldots (13)$$

$$\bar{C}_{31}^{-1,3} = \bar{C}_{31,31} - 0.825\,\bar{C}_{33,31} - 0.967\,\bar{C}_{35,31} - 0.513\,\bar{C}_{37,31} + 0.049\,\bar{C}_{39,31}$$

$$+ 0.445\,\bar{C}_{41,31} + 0.571\,\bar{C}_{43,31} + 0.452\,\bar{C}_{45,31} + \cdots ,$$

$$\ldots\ldots (14)$$

with similar equations for S .

The error in geoid height implied by the standard deviations in equations (12) may be assessed by the method of section 4.2, with $\bar{Q} = 2.283$ for $(q,k) = (1,1)$ and $\bar{Q} = 1.563$ for $(q,k) = (-1,3)$. The resulting accuracies in geoid height are 7 cm for C and 2 cm for S , with $(q,k) = (1,1)$; and 4 cm for C and 7 cm for S with $(q,k) = (-1,3)$. The average is 5 cm, and this is very satisfactory as the average from Samos 2, with drag half that of 1982-07A, was 4 cm.

6 CONCLUSIONS

The 26 orbits of Cosmos 1335, determined from US Navy, Hewitt camera and visual observations at the time of 31:2 resonance, have proved accurate enough to evaluate even-degree lumped harmonics of order 31 with an average accuracy equivalent to about 1.5 cm in geoid height, and odd-degree lumped harmonics with an accuracy between 2 and 7 cm. These results are for a new inclination, 74°, and should make a valuable contribution to the determination of individual harmonic coefficients of order 31.

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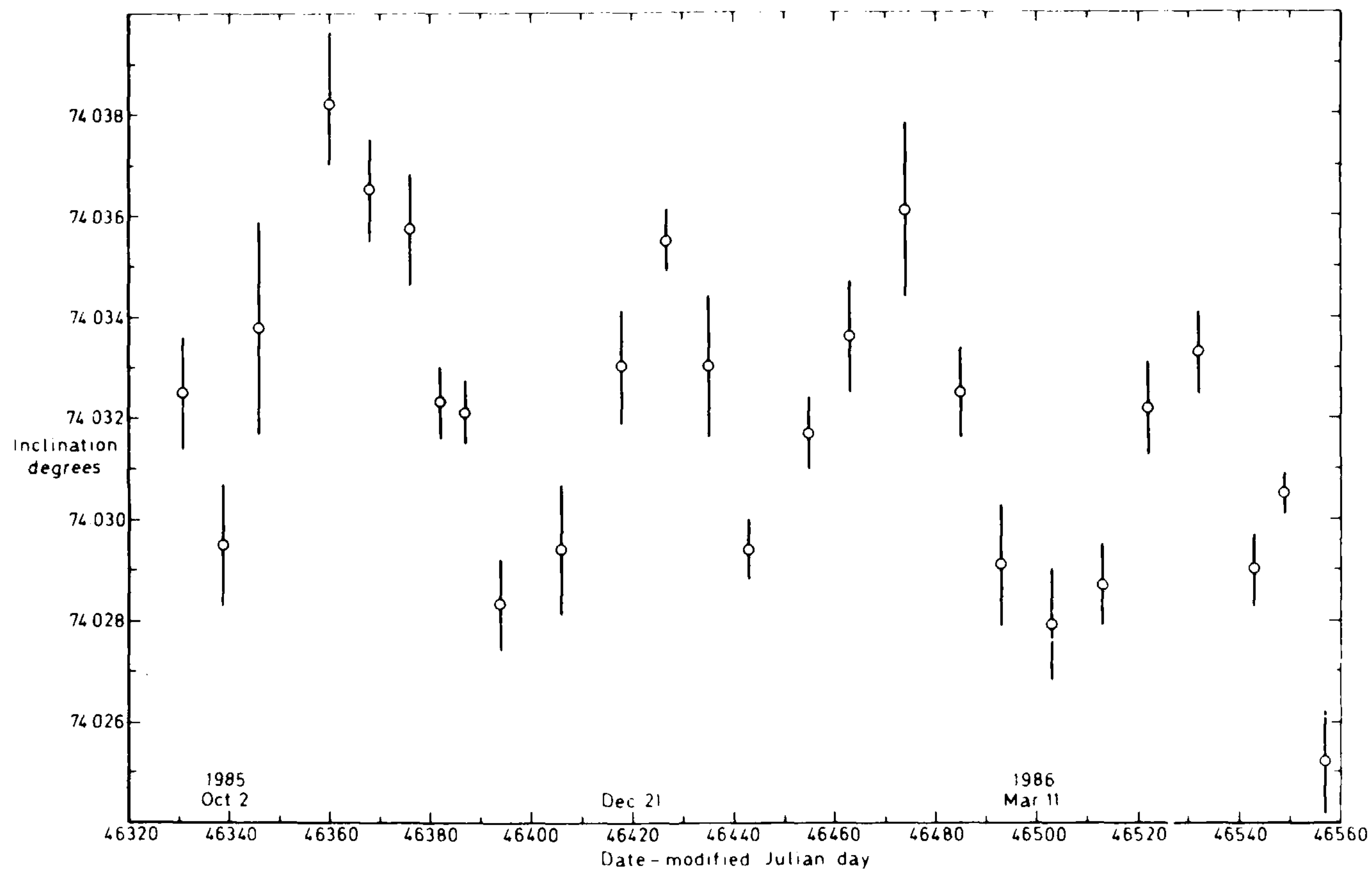

Fig 1 Values of inclination from Table 2, with standard deviations

Fig 2

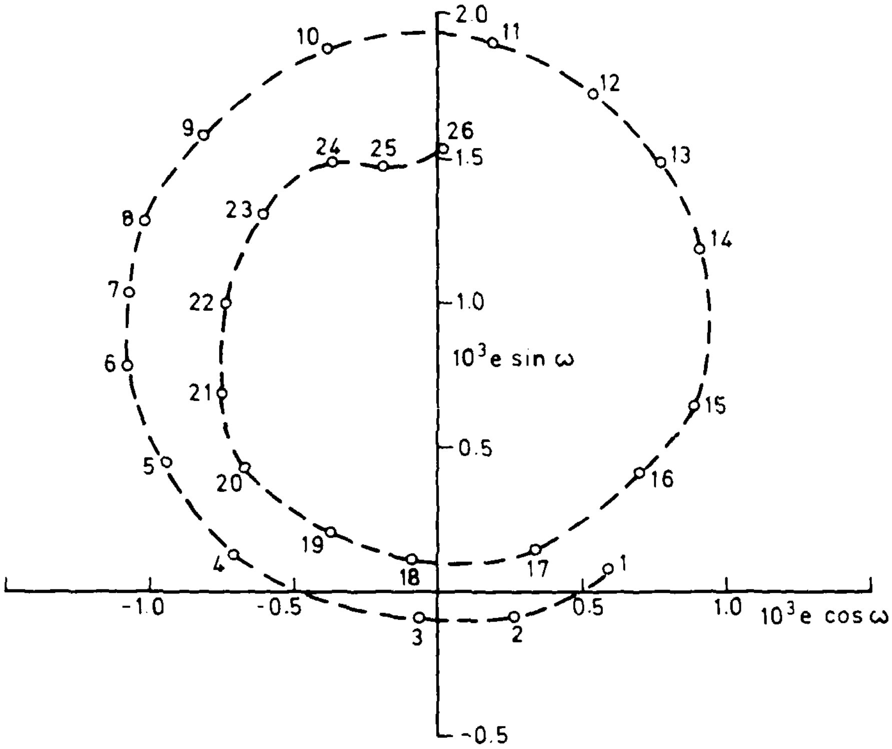

Fig 2 Values of e and ω from the 26 PROP orbits: polar diagram

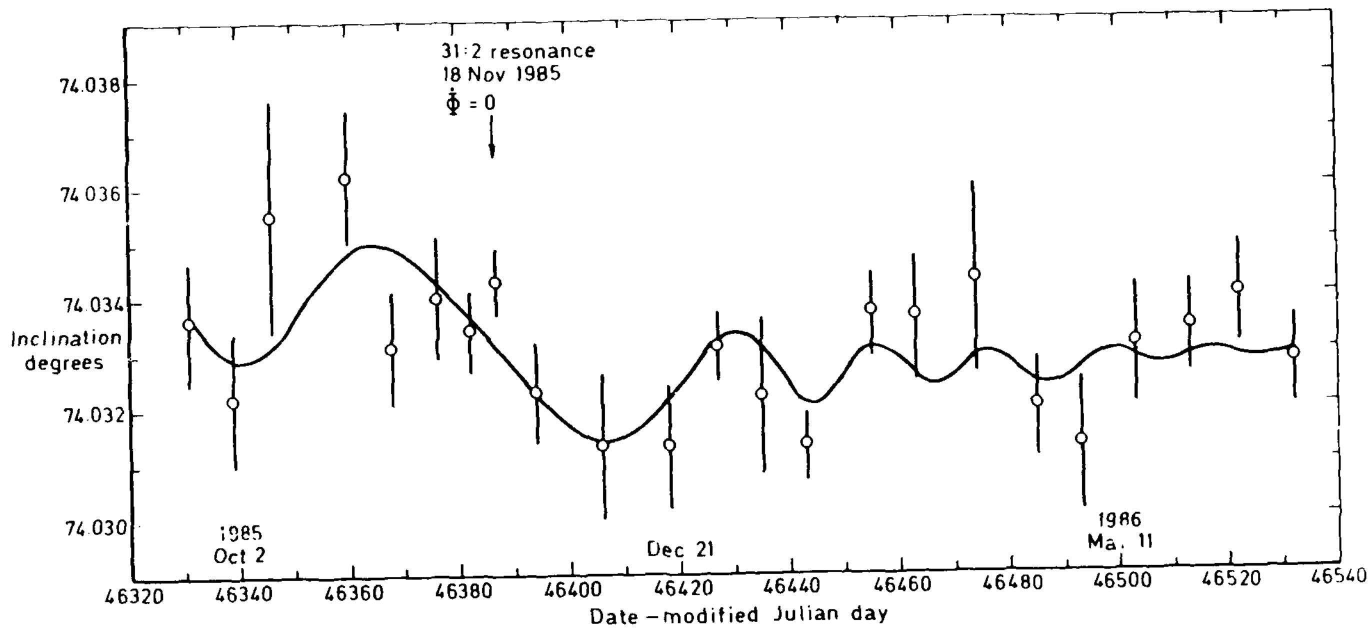

Fig 3 Values of inclination after removal of perturbations, with curve fitted by THROE

Fig 3

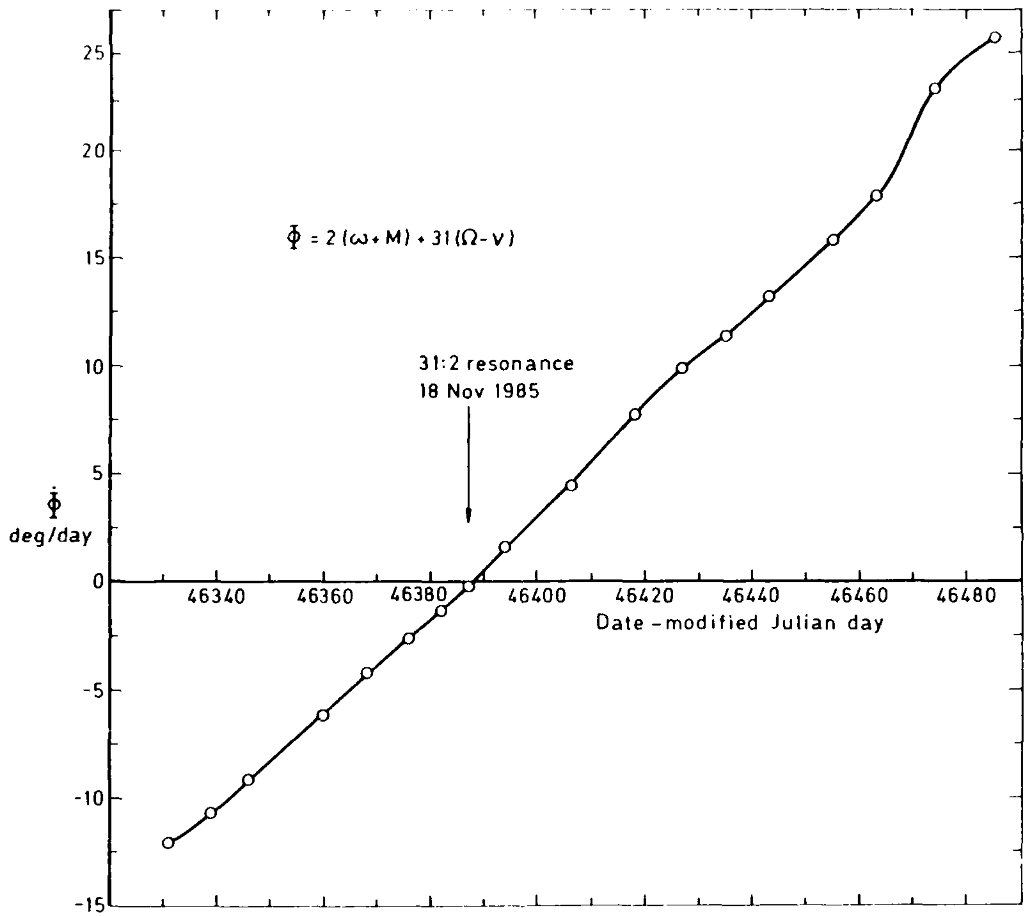

Fig 4 Variation of $\dot{\Phi}$

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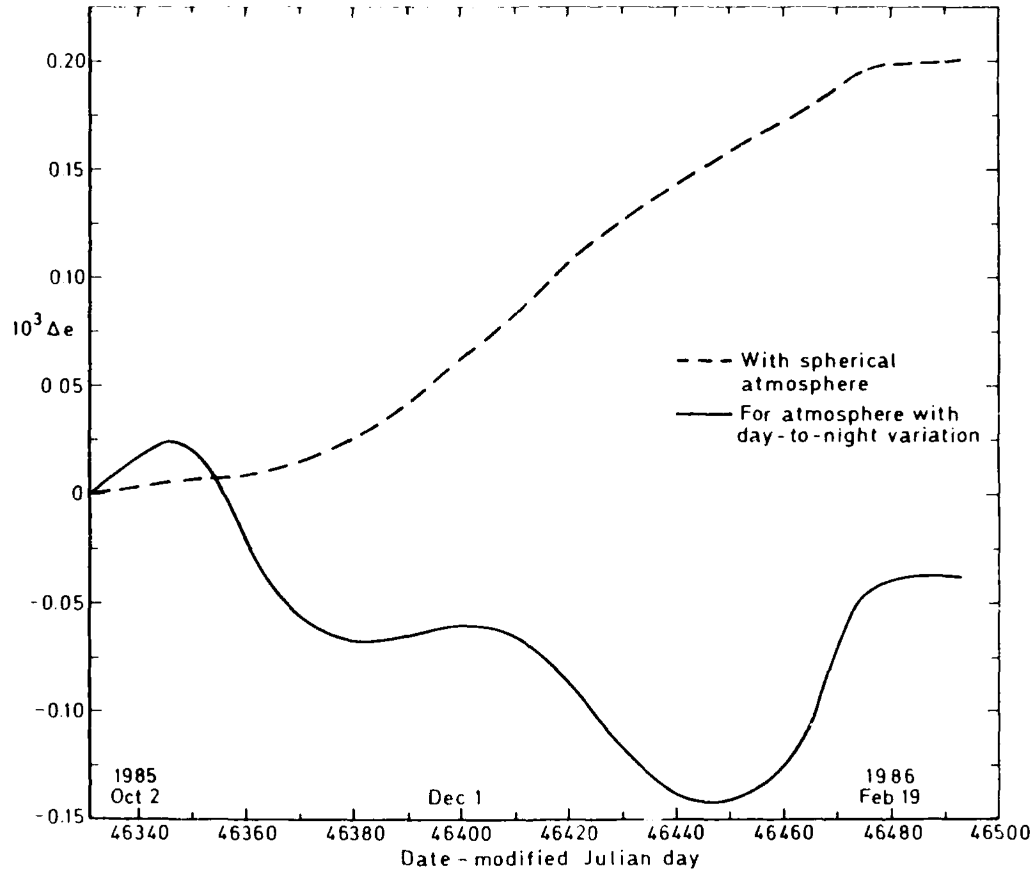

Fig 5 Atmospheric corrections Δe applied to eccentricity

Fig 6

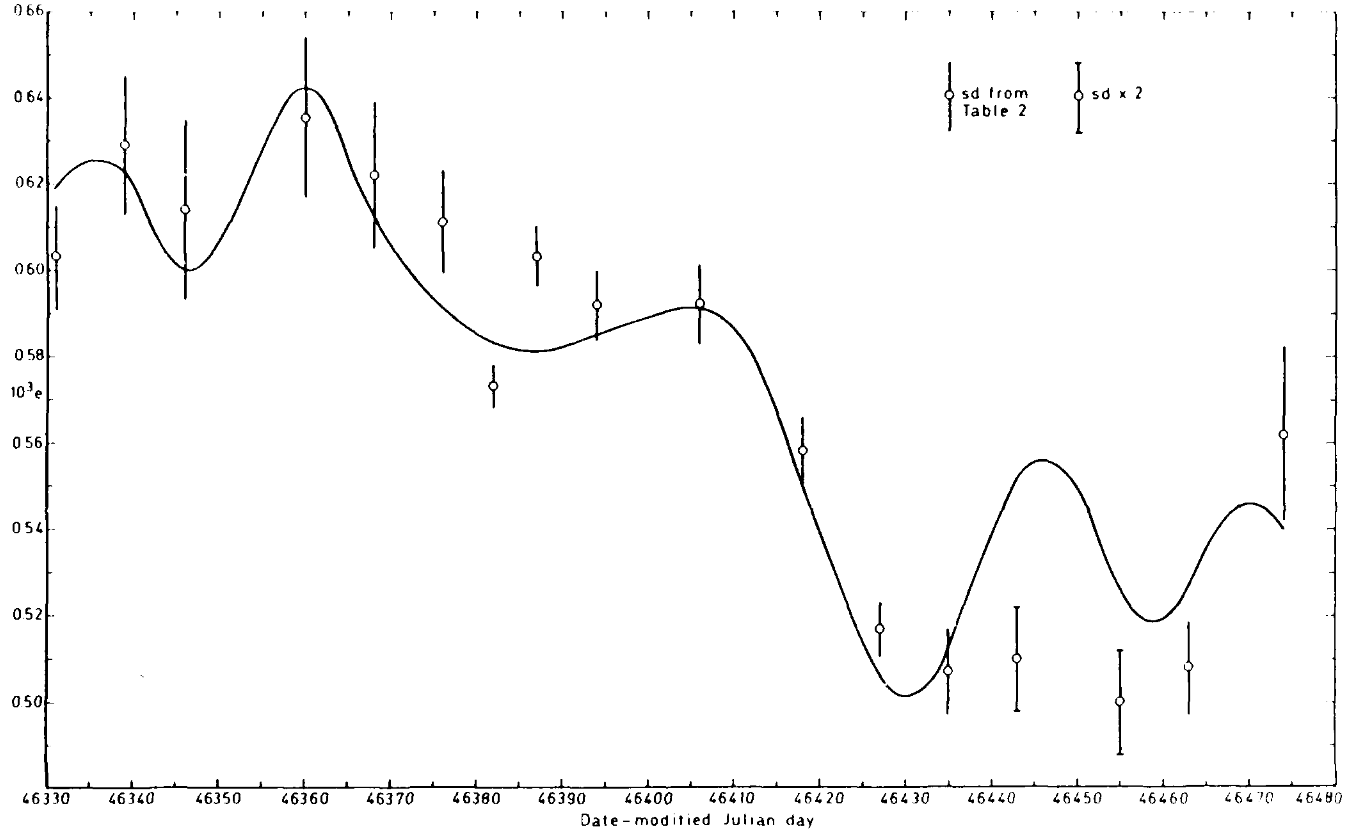

Fig 6 Values of eccentricity, after removal of perturbations (including atmosphere with day-to-night variation), with fitted curve for $(\gamma, q) = (1,1)$ and $(1,-1)$

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17. Abstract

Cosmos 1335 (1982-07A) was launched on 29 January 1982 into a near-circular orbit of inclination 74°, and decayed on 5 April 1987. The orbit has been determined from observations for 26 epochs between September 1985 and May 1986, a time when the orbit was experiencing the effects of 31:2 resonance with the Earth's gravitational field. About 1400 observations were used, the most numerous being those from the US Navy Navspasur system, and the most accurate those from the Hewitt cameras of the University of Aston sited at Herstmonceux and at Siding Spring in Australia. The average orbital accuracy achieved was about 70 m radial and 120 m cross-track.

Analysis of the changes in inclination and eccentricity at resonance has yielded useful values for six lumped harmonics in the geopotential of order 31: the two lumped harmonics of even degree had an average accuracy equivalent to 1.5 cm in geoid height; the four of odd degree had accuracies between 2 and 7 cm. Great Britain,

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